TRINITY COLLEGE

MATHEMATICS SPECIALIST UNIT 1
Section Two:
Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

Student number: In figures

In words   _____

Your name   _____

Time allowed for this section
Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

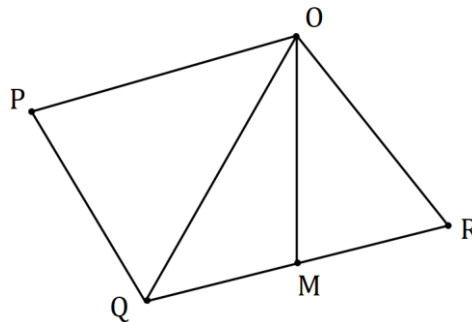**Section Two: Calculator-assumed** 65% (98 Marks)

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 (5 marks)

In the diagram below, M is the midpoint of QR.

If $\overrightarrow{OP} = \mathbf{p}, \overrightarrow{OQ} = \mathbf{q}$ and $\overrightarrow{OR} = \mathbf{r}$, express the following in terms of $\mathbf{p}, \mathbf{q}$ and $\mathbf{r}$.

(a) $\overrightarrow{QP}$. (1 mark)

(b) $\overrightarrow{OM}$. (2 marks)

(c) $8\overrightarrow{MP}$. (2 marks)

Question 10   **(8 marks)**

Points P, Q and R have coordinates $(-2, 11), (8, 15)$ and $(17, 3)$ respectively. Determine

(a) $\overrightarrow{PQ}$.   (1 mark)

(b) $|\overrightarrow{QR}|$.   (2 marks)

(c) $2\overrightarrow{PQ} - 60\mathbf{u}$, where $\mathbf{u}$ is a unit vector in the direction $\overrightarrow{QR}$.   (3 marks)

(d)   The coordinates of point S, given that $\overrightarrow{RS} = \overrightarrow{QP}$.   (2 marks)
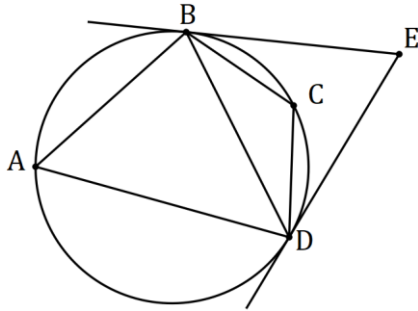
Question 11   **(8 marks)**

(a)   Show that the vectors $(-10, 5)$ and $(4.5, 9)$ are perpendicular.   (2 marks)

(b)   Determine, to the nearest degree, the angle between the vectors $(1, -5)$ and $(2, -3)$.   (2 marks)

(c)   The vectors $(a, 2a - 1)$ and $(a - 4, 3)$ are perpendicular, where a is a constant. Determine the value(s) of a and the corresponding pair(s) of vectors.   (4 marks)

Question 12 **(7 marks)**

(a) In the diagram below (not drawn to scale) A, B, C and D lie on a circle and EB and ED are tangents to the circle. If $\angle BED = 54°$ and $\angle CDB = 20°$, determine the size of $\angle CBD$.

(3 marks)

(b) Quadrilateral $ABCD$ is such that $CB = CD$, $\angle BAD = 96°$ and $\angle BDC = 48°$.

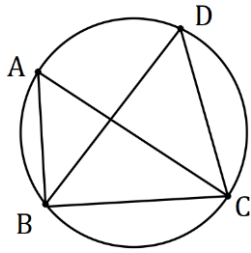(i) Sketch a diagram to show this information. (1 mark)

(ii) Show that $ABCD$ is cyclic and hence determine the size of $\angle CAD$. (3 marks)
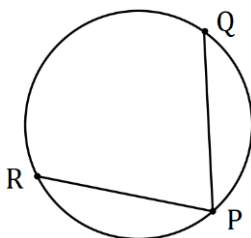
Question 13 (8 marks)

(a) A, B, C and D lie on a circle with diameter AC (diagram not to scale). Determine the size of $\angle BDC$ when $\angle BCA = 40°$. (2 marks)

(b) K, L and M lie on a circle (diagram not to scale). Secant KN cuts the circle at L and JN is a tangent to the circle at M. Given that $\angle LNM = 33°$ and $\angle LMN = 43°$, determine the size of $\angle MKL$ and the size of $\angle KMJ$. (3 marks)
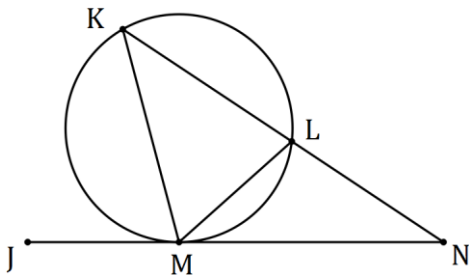
(c) P, Q and R lie on a circle of radius 54 mm (diagram not to scale) and $PQ = PR = 68$ mm. Determine the size of angle $\angle QPR$, to the nearest degree. (3 marks)

Question 14 **(9 marks)**

The parts of this question refer to the word DECOMPOSITION. It has 7 different consonants and 6 vowels, some of which are repeated.

(a) Determine the number of ways that 4 different consonants chosen from the letters of the word can be arranged in a row. (1 mark)

(b) Determine the number of ways that all the letters of the word can be arranged in a row. (2 marks)

(c) Determine the number of ways that all the letters of the word can be arranged in a row if the consonants must all be adjacent. (3 marks)

(d) Determine how many 3 letter permutations (e.g. MTE, ONO, etc) can be made using the letters of the word. (3 marks)
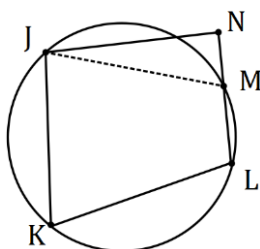
Question 15   (8 marks)

(a)   In the diagram below (not drawn to scale) P, Q, R and S lie on the circle with centre O.
       Determine the size of angles α, β and γ given that $\angle PQR = 128°$ and $5\beta = 3\gamma$.   (4 marks)
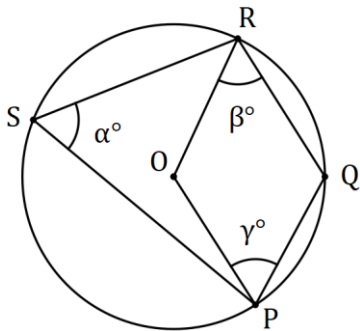
(b)   Write the converse of the theorem that states the opposite angles of a cyclic quadrilateral
       are supplementary.   (1 mark)

(c)   Prove by contradiction that the converse you wrote in (b) is true. Start by assuming that
       there is a quadrilateral that *does* have supplementary opposite angles but is *not* cyclic,
       such as $JKLN$ shown below.   (3 marks)

See next page

Question 16 **(7 marks)**

Airport P lies 155 km away from airport Q on a bearing of 072°. A plane leaves airport P at 9:30 am to fly to airport Q. The plane can maintain a speed of 195 kmh⁻¹ and there is a steady wind of 25 kmh⁻¹ blowing from the south.

Determine the bearing that the helicopter should steer and the time of its arrival at airport Q, to the nearest minute.

Question 17 **(7 marks)**

Three forces **a**, **b** and **c** act on a point in a plane.

The forces are $\mathbf{a} = -44\mathbf{i} + 66\mathbf{j}$ N, $\mathbf{b} = -12\mathbf{i} - 75\mathbf{j}$ N and $\mathbf{c} = 180\mathbf{i} + 102\mathbf{j}$ N.

(a)   Determine the magnitude of the resultant force and the direction, to the nearest degree, that the resultant makes with the vector **i**. (3 marks)

When $\lambda\mathbf{a} + \mu\mathbf{b} + \mathbf{c} = 0$, the forces are in equilibrium.

(b)   Determine the values of the scalar constants λ and μ for equilibrium to occur.   (4 marks)

Question 18   **(8 marks)**

(a)   A set of cards is numbered with all the odd numbers between 101 and 999. Determine the
       minimum number of cards that must be selected to ensure that at least 6 cards in the
       selection have the same last digit. Justify your answer using the pigeonhole principle.

                                                                                  (3 marks)

(b)   Nine different books sit on a shelf, one of which is non-fiction and the rest fiction. A
       student is told they can take away as many of them as they like but must not leave empty
       handed. Determine how many different selections can be made

       (i)   of exactly 6 books.   (1 mark)

       (ii)   altogether.   (2 marks)

       (iii) that include the non-fiction.   (2 marks)

Question 19   **(8 marks)**

Determine how many of the integers between 1 and 500 inclusive are

(a)   divisible by 7.   (1 mark)

(b)   divisible by 7 or 9.   (3 marks)

(c)   divisible by 7 or 9 but not both.   (1 mark)

(d)   divisible by 7 or 9 but not 6.   (3 marks)

Question 20   **(7 marks)**

$ABCD$ is a trapezium with $\overrightarrow{AB}$ parallel and in the same direction to $\overrightarrow{DC}$.

(a)   Sketch a labelled diagram of $ABCD$.   (1 mark)

(b)   Show that $\overrightarrow{DB} + \overrightarrow{AC} = \overrightarrow{DC} + \overrightarrow{AB}$.   (2 marks)

(c) M lies on AC and N lies on BD so that $AM{:}MC = BN{:}ND = 2{:}3$. Use a vector method to prove that $ABNM$ is a trapezium.   (4 marks)

Question 21 **(8 marks)**

A helicopter, with a maximum speed through still air of 240 km/h, leaves its base at A to fly to a destination at B, where $\overrightarrow{AB} = (-45\mathbf{i} + 80\mathbf{j})$ km.

There is a steady wind of velocity $(-\mathbf{i} - 3\mathbf{j})$ km/h.

The velocity that the pilot of the helicopter must set to travel from A to B is $(a\mathbf{i} + b\mathbf{j})$, where a and b are constants.

(a)   Determine the velocity vector the helicopter pilot should set in order to fly directly from A to B in the shortest amount of time.   (6 mark)

(b)   What is the shortest journey time, to the nearest minute?   (2 marks)

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Supplementary page

Question number: _____